Estimation and empirical performance of non-scalar DCC models

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Contributions

1. A new method for estimating the parameters of DCC models with non-scalar specifications. This method is applied to the Hadamard version of the DCC model with parameter matrices of full rank, rank equal to 2, and rank equal to 1.
2. A new Almon specification of the rank 1 DCC model: functional parametrization with fixed parameter space dimension (as in the scalar DCC) but more flexible than the standard scalar model.
3. Empirical results show that the use of more richly parametrized models adds performance with respect to the scalar case according to several in- and out-of-sample statistical tests.

Hadamard DCC: general case

Model setup:

- $\mathbf{r}_t = H_t^{1/2}\boldsymbol{\xi}_t, \ \{\boldsymbol{\xi}_t\} \sim \mathrm{IN}(\mathbf{0}, \mathbf{I}_n)$.
- The conditional covariance process H_t is modeled as
$$H_t = D_t R_t D_t,$$
with $D_t := \mathrm{diag}(\sigma_{1,t}, \ldots, \sigma_{n,t})$ and $\sigma^2_{i,t}$ the outcome of a GARCH model estimation.
- The conditional correlation process R_t is given by:
$$R_t = \mathrm{Diag}(Q_t)^{-1/2}\, Q_t\, \mathrm{Diag}(Q_t)^{-1/2},$$
with
$$Q_t = (\mathbf{i}_n\mathbf{i}_n^T - A - B) \odot S + A \odot (\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^T) + B \odot Q_{t-1},$$
where $\boldsymbol{\varepsilon}_t := D_t^{-1}\mathbf{r}_t$, $\mathbf{i}_n = (1, \ldots, 1)^T \in \mathbb{R}^n$, $\odot$ denotes the Hadamard product, parameters $A, B \in \mathbb{S}_n$ are symmetric matrices of dimension n, and $S \approx \mathrm{E}\left[\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^T\right]$ can be "targeted".

Parameter constraints:

- **(SC) Stationarity constraints:** $|A_{ij} + B_{ij}| < 1, \ i \in \{1, \ldots, n\}, \ j \in \{1, \ldots, n\}$.
- **(PC) Positivity constraints:** $A \succeq 0, \quad B \succeq 0, \quad (\mathbf{i}_n\mathbf{i}_n^T - A - B) \odot S \succ 0$.

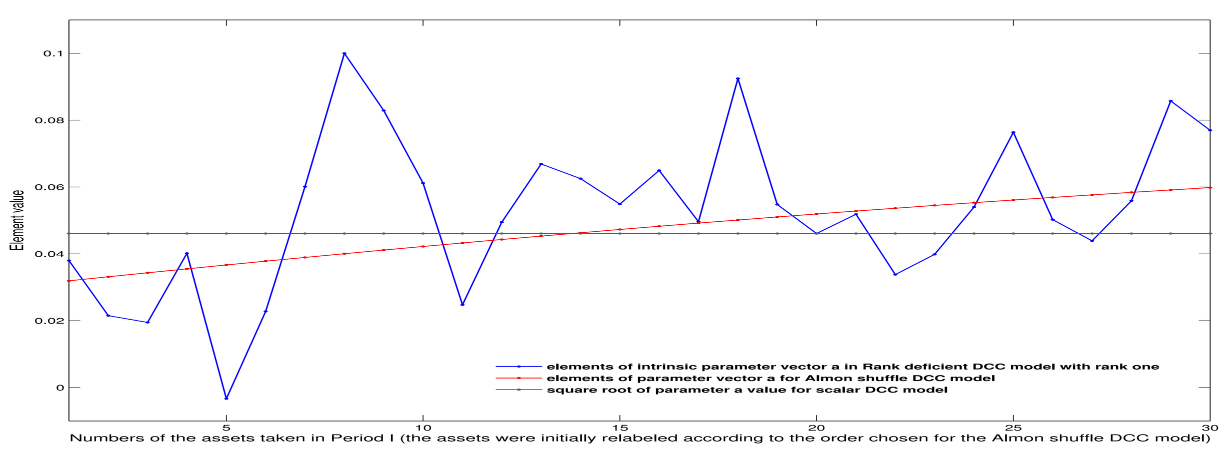
Hadamard DCC: special cases

- Full rank **Hadamard** model: $A, B \in \mathbb{S}_n$ have rank $r = n$.
- **Rank 2** deficient model: $A, B \in \mathbb{S}_n$ have rank $r = 2$; in this case parameter matrices are defined via $A := \tilde{A}\tilde{A}^T$, with $\tilde{A}$ a matrix of dimension $n \times 2$, $\mathrm{vec}(\tilde{A}) \in \mathbb{R}^{2n-1}$.
- **Rank 1** deficient model: $A, B \in \mathbb{S}_n$ have rank $r = 1$; in this instance parameters are written as $A := \boldsymbol{a}\boldsymbol{a}^T$, with $\boldsymbol{a} \in \mathbb{R}^n$.
- **Almon** models: particular case of the rank 1 deficient model with A and B parametrized using the Almon-lag function.
- **Scalar** model: in this case $A := a\mathbf{i}_n\mathbf{i}_n^T$ and $B := b\mathbf{i}_n\mathbf{i}_n^T$, with $a, b \in \mathbb{R}$.

(SC) and **(PC)** for each specification are provided in [1].

Almon DCC and Almon Shuffle DCC models

- It is a particular case of the rank 1 deficient model: each of the parameters $A, B \in \mathbb{S}_n$ of rank $r = 1$ is provided as $A := \tilde{\boldsymbol{a}}\tilde{\boldsymbol{a}}^T$, with $\tilde{\boldsymbol{a}} := \mathrm{alm}_n(\boldsymbol{a}) \in \mathbb{R}^n$ and $\boldsymbol{a} \in \mathbb{R}^3$.
- Almon operator: $\mathrm{alm}_n : \mathbb{R}^3 \to \mathbb{R}^n$, with
$$(\mathrm{alm}_n(\boldsymbol{a}))_i := a_1 + \exp(a_2 i + a_3 i^2), \ \ i \in \{1, \ldots, n\}.$$
It transforms any given vector $\boldsymbol{a} \in \mathbb{R}^3$ into a vector of length n whose entries are the values of the exponential Almon function. For example:



- In the Almon Shuffle (Almon S.) model the assets are first ordered in decreasing order according to the magnitude of their projection on the first principal component (carried out using the unconditional covariance matrix of the returns).

Model hierarchy and numbers of parameters

$$\begin{array}{ccccccc} & & & & \text{Almon} & & \\ & & & & \downarrow & \nwarrow & \\ \text{Hadamard} & \longleftarrow & \text{Rank 2 deficient} & \longleftarrow & \text{Rank 1 deficient} & \longleftarrow & \text{Scalar} \\ & & & & \uparrow & \swarrow & \\ & & & & \text{Almon Shuffle} & & \end{array}$$

	n	5	10	15	20	25	30
Hadamard	$n(n+1)$	30	110	240	420	650	930
Rank 2 deficient	$2(2n-1)$	18	38	58	78	98	118
Rank 1 deficient	$2n$	10	20	30	40	50	60
Almon	6	6	6	6	6	6	6
Scalar	2	2	2	2	2	2	2

Algorithm: maximize the Gaussian quasi-LF under (SC)-(PC)

- The optimization method is based on recursively minimizing penalized local models that incorporate Bregman divergences ensuring that the constraints are satisfied.
- We provide in [1] the explicit expressions of all the divergences, the local models, their gradients, and Jacobians.

Data (30 components of DJIA in 2013) and setup

- Daily returns 01/19/96-12/21/10: first 3000 observations (ending 12/31/07) for estimation and remaining 750 for out-of-sample testing.
- Actual estimation data are OLS residuals of a CAPM one-factor model, estimated using the 3000 initial observations, and then 750 out-of-sample CAPM adjusted returns are computed.
- Out-of-sample forecasts of the H_t matrices of each DCC model are computed using only the in-sample observations.

Summary of estimated A matrices of DCC models

n		Scalar	Hadamard	Rank 1	Rank 2	Almon	Almon S.
5	mean	0.0040	0.0083	0.0030	0.0042	0.0048	0.0046
	std	-	0.0078	0.0060	0.0071	0.0015	0.0069
10	mean	0.0046	0.0056	0.0076	0.0044	0.0050	0.0045
	std	-	0.0046	0.0055	0.0037	0.0011	0.0016
15	mean	0.0036	0.0038	0.0039	0.0032	0.0036	0.0036
	std	-	0.0038	0.0026	0.0025	0.0004	0.0015
20	mean	0.0026	0.0025	0.0029	0.0025	0.0026	0.0026
	std	-	0.0024	0.0015	0.0020	0.0001	0.0006
25	mean	0.0024	0.0023	0.0033	0.0029	0.0023	0.0024
	std	-	0.0023	0.0020	0.0021	0.0000	0.0004
30	mean	0.0021	0.0019	0.0028	0.0024	0.0021	0.0022
	std	-	0.0023	0.0018	0.0012	0.0001	0.0006

Table 2. Mean and standard deviation of all the entries of the estimated matrices A.

In-sample results (AIC and LR tests)

n		Scalar	Almon	Almon S.	Rank 1	Rank 2	Hadamard
5	$\mathrm{AIC}^{\mathrm{rank}}$	-27.9665^3	-27.9644^4	-27.9683^2	**-27.9685^1**	-27.9643^5	-27.9591^6
	p-value	—	0.7736	**0.0085**	**0.0049**	0.0603	0.2032
10	$\mathrm{AIC}^{\mathrm{rank}}$	-56.5862^2	-56.5843^3	-56.5835^4	**-56.5870^1**	-56.5807^5	-56.5392^6
	p-value	—	0.6537	1.0000	**0.0034**	**0.0191**	0.9933
15	$\mathrm{AIC}^{\mathrm{rank}}$	-84.0659^3	-84.0637^5	-84.0680^2	**-84.0684^1**	-84.0653^4	-83.9550^6
	p-value	—	0.8443	**0.0067**	**0.0001**	**0.0000**	1.0000
20	$\mathrm{AIC}^{\mathrm{rank}}$	-113.2773^4	-113.2753^5	-113.2786^3	**-113.2818^1**	-113.2790^2	-113.0702^6
	p-value	—	0.7419	**0.0183**	**0.0000**	**0.0000**	1.0000
25	$\mathrm{AIC}^{\mathrm{rank}}$	-142.3120^3	-142.3098^4	-142.3129^2	**-142.3137^1**	-142.3062^5	-141.9760^6
	p-value	—	0.8475	**0.0295**	**0.0000**	**0.0000**	1.0000
30	$\mathrm{AIC}^{\mathrm{rank}}$	-171.7851^4	-171.7866^3	-171.7894^2	**-171.8047^1**	-171.7811^5	-171.3381^6
	p-value	—	**0.0139**	**0.0004**	**0.0000**	**0.0000**	1.0000
	$\mathrm{AIC}^{\mathrm{rank}}$	19^3	24^4	15^2	**6^1**	26^5	36^6

Table 3. LR test p-values with null the scalar model against the other parametrizations.

Out-of-sample results

Model Confidence Set (MCS) test: under the correct specification, the deGARCHed returns $\boldsymbol{\varepsilon}_t := D_t^{-1}\mathbf{r}_t$ have the specified DCC correlation matrix $R_t = (\rho_{ij,t})$; we use the loss function
$$d_t := \frac{2}{n(n-1)} \sum_{i<j=2,\ldots,n} (\varepsilon_{i,t}\varepsilon_{j,t} - \rho_{ij,t})^2.$$

n		Scalar	Hadamard	Rank 1	Rank 2	Almon	Almon S.
5	Position	5	**4**	**1**	**3**	6	**2**
	p-value	0.008	0.073	1.000	0.228	0.005	0.228
10	Position	**5**	**3**	**1**	**4**	**6**	**2**
	p-value	0.190	0.433	1.000	0.260	0.190	0.433
15	Position	**3**	**5**	**4**	**6**	**2**	**1**
	p-value	0.242	0.127	0.242	0.061	0.811	1.000
20	Position	3	6	5	4	2	**1**
	p-value	0.003	0.003	0.003	0.003	0.026	1.000
25	Position	3	6	5	4	**2**	**1**
	p-value	0.002	0.000	0.001	0.002	0.074	1.000
30	Position	3	6	4	5	**2**	**1**
	p-value	0.006	0.000	0.006	0.001	0.394	1.000
	Score	22	30	20	26	20	**8**

Table 4. For each n, MCS at 5% are given in red; MCS at 10% in bold black.

Main findings and extensions

- Estimation of non-scalar DCC models subjected to (SC)-(PC) is feasible in dimensions up to 30 (at least).
- Non-scalar models show good performance with respect to both in-sample and out-of-sample criteria; the scalar model fails the MCS test for higher dimensions.
- In progress: same approach for DVEC models, and using composite likelihood estimation.

References

[1] Luc Bauwens, Lyudmila Grigoryeva, and Juan-Pablo Ortega. Estimation and empirical performance of non-scalar dynamic conditional correlation models. 2014.

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