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Maple-exploring of a Free Flywheel Suspended by Superconductive Bearing

The software Maple as a powerful tool to analyze complicated non-linear dynamic systems is used to explore parametrical conditions guaranteeing the stability of resting or rotating free magnetically levitated flywheel's rotor, to solve the Cauchy problem and to build the phase portraits. The dynamical model is derived on the basis of analytical electromechanics with taking into account six degrees of freedom of a free rotor, the constancy of full magnetic fluxes coupled with immobile and rotor's superconducting rings, Lyapunov stability theorems, and Euler equations of a free body dynamics.

Key words: superconducting levitation, free flywheel, dynamics.

Григор'єва Л.В., аспірантка

Maple- дослідження вільного ротора на надпровідних підшипниках

Система комп'ютерної математики Maple як засіб аналізу складних нелінійних динамічних систем застосована для дослідження параметричних умов, що гарантують стійкість вільного магнітно левітуючого ротора у стані рівноваги або при обертанні, для розв'язання задачі Коші і для побудови фазових портретів. Динамічна модель отримана на основі аналітичної електромеханіки з врахуванням шести степенів свободи вільного ротора, сталості повного магнітного потоку, зв'язаного з нерухомими надпровідними кільцями та кільцями ротора, теорем Ляпунова про стійкість та динамічних рівнянь Ейлера для вільного тіла.

Ключові слова: надпровідна левітація, вільний маховик, динаміка.

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I. Introduction

Magnetic bearings become a very important element for renewable energy sources, end-use energy efficiency, environmentally preferred advanced generation, and flywheel energy storage. As it is expected, devices using magnetic bearings can be widely implemented, from automobile or helicopter engines to power reserve flywheels to level peak energy consumptions.

We developed a new type of magnetic bearings based on the "Magnetic Potential Well" (MPW) phenomenon [1, 2]. For two constantly oriented closed superconducting loops, the MPW-manifestation signifies that with nearing of these spaced loops, their magnetic attractive force does not increase as usually but decreases, becomes zero and changes into the repulsive magnetic force before the spacing between loops equals zero. A similar picture can be observed in a permanent magnet-closed zero resistant loop pair. The MPW can be also realized in a system with many magnets.

Magnetic bearings have many advantages over ball, gas, and hydro bearings: practically unlimited operating time, absence of a lubricant, simplicity and reliability of operation, etc. But many applications of magnetic bearings require solving some problems. One of them is the stability of the free resting or rotating rotor. Trials to suspend a body in the free equilibrium under action of magnetic forces are fruitless on the basis of Earnshaw's theorem [3]. There are only three exceptions not covered by this theorem. One of them is automation allowing transforming an unstable dynamic system into the stable system by e.g. feed-back control. The second is diamagnetism of a substance particularly bulk superconductors [4 - 6]. And finally, the MPW based on zero resistance of a closed current carrying loop [1, 2]. It should be noted that any of these exceptions assists but doesn't guarantee the free body rest or rotation stability.

Not only stability but also centring force levels are important in bearing applications. The stiffness of magnetic bearing must be on the level of commercial bearings operating with radial stiffness no less than 10^6-10^7 N/mm. Possibilities to satisfy these levels by known magnetic bearings do not look optimistic. Really, the top pressure of "warm" magnetic bearings with automation is restricted from above app. by $2 \cdot 10^6$ N/m^2 determined by the magnetic saturation induction of 2-2.5 Tesla. For characteristic bearing size of 0.1 m it gives magnetic bearing stiffness of $2 \cdot 10^5$ N/m that corresponds with published data [7].

The top pressure of 10^3-10^4 N/m^2 for passive superconducting bearings based on the ideal diamagnetism is restricted by the first critical magnetic field, which is less than 0.2 Tesla for all known superconductors. This is too low value to be estimated as practicable.

In contrast to known magnetic bearings, instead of relying on the superconductor diamagnetism or automatic control to keep the air gap within required limits, we propose to use the MPW phenomenon. Manifestation of this phenomenon requires ideal electric conductivity in a closed loop of any shape. This allows using existing high current density superconductors e.g. niobium-titanium, niobium-germanium, and niobium-tin joining. Superconducting magnets using these superconductors and operating in the persistent current mode are in abundance. They are capable of generating constant magnetic field no less than 10-20 Tesla without electric losses that is unreachable in all other magnetic bearings. Before MPW these magnets were used as high magnetic field sources only. Now their applications can be MPW-bearings with pressure of 10^8 N/m^2 and stiffness of 10^7 N/m.

II. Flywheel Model

A. Coordinate Systems

A flywheel is modeled by two stator's coaxial immobile superconductive rings magnetically interacting with two sets of N small planar superconductive circular loops (dipoles) equally shifted in relation to the rotor's axis (see Figure 1). The MPW is realized in a set of N dipoles and the nearer superconductive immobile ring. It means that at some rotor/stator coaxiality position, the electric current in each dipole is zero because its full magnetic flux (magnetic linkage) that is constant on the basis of the Faraday's electromagnetic induction law is created by the nearer immobile ring, yet at any another rotor/stator geometrical configuration this current is non-zero to satisfy the magnetic linkage constancy.

To determine the potential energy of magnetic interaction, it is convenient to introduce some trihedrons. The origin O of the immobile trihedron $Oxyz$ with unit vectors $\vec{i}_1, \vec{i}_2, \vec{i}_3$ is the symmetry center of two coaxial superconductive rings of radius a, and the vertical rings' symmetry axis Oz is parallel to the gravity force.

The origin O_1 of the second trihedron $O_1x_1y_1z_1$ with unit vectors $\vec{i}'_1, \vec{i}'_2, \vec{i}'_3$ coincides with a free rotor mass center so that axis O_1z_1 is directed along the rotor's symmetry axis. As an example, the case $N = 4$ is shown in Figure 1. The free rotor is described by six degrees of freedom. The first three are Cartesian $Oxyz$-coordinates x, y, z of the point O_1. Non-dimensional linear coordinates X, Y, and Z are derived by division of a corresponding Cartesian $Oxyz$-coordinate by value a. The other three degrees of freedom are the Euler-Krylov angles x_4, x_5, x_6 determining the rotor space orientation and x_4 is rotation relatively axis O_1x_1, x_5 is rotation relatively axis O_1y_1, and x_6 is rotation relatively axis O_1z_1 at that.

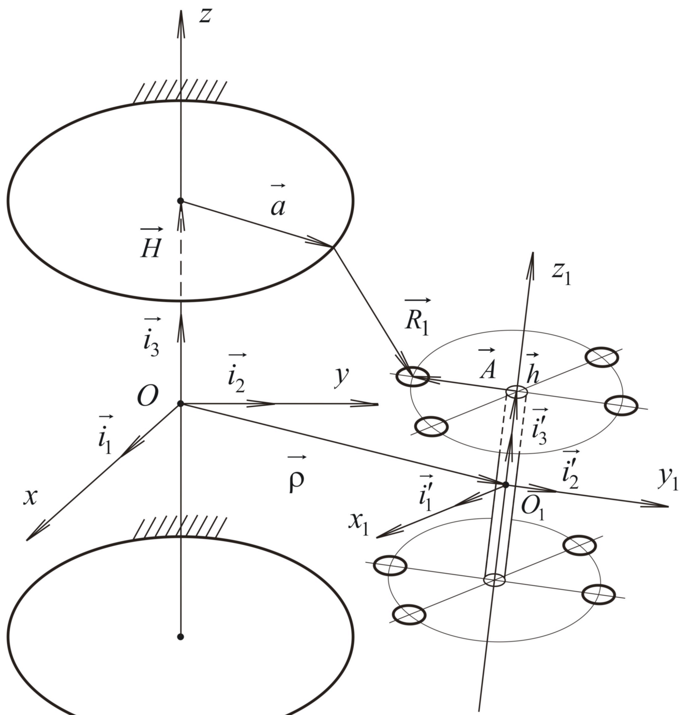

Fig. 1. Sketch of the flywheel.

B. Potential Energy of Gravity and Magnetic Interactions

The potential energy U of the flywheel dynamic system consists of two parts. One is the gravity represented by the rotor's constant weight force. The second is the sum of magnetic interactions of each j-immobile superconducting ring (j =1,2) and N dipoles of upper or lower part of the rotor. Magnetic interactions between dipoles are ignored as infinitesimal. Dipoles are supposed to be zero resistance closed loops with taking into account the constancy of their magnetic linkages as a consequence of zero resistance of each dipole's loop.

The MPW-position that is at our disposal corresponds to the coincidence of axes Oz and O_1z_1. At this coaxiality position, the rotor's weight $G = mg$ where m is its mass and g is the gravity acceleration can be balanced at $Z = 0$ by the difference in attractive forces of the upper part of superconducting loops (one immobile and N on the rotor top) directed above and the lower similar part directed downward. Such coaxial equilibrium can be accomplished e.g. by the adjusted parameter $k = \Psi_2\Psi_1^{-1} = const > 1$ that is ratio between the upper immobile superconducting ring magnetic linkage Ψ_2 and lower ring linkage Ψ_1.

Below we use the matrix M determining a relative space orientation of the trihedrons $Oxyz$ and $O_1x_1y_1z_1$ (see e.g. [8], p.119) in the expanded form

$$M = \begin{pmatrix} c_5c_6 & c_4s_6 + s_4s_5c_6 & s_4s_6 - c_4s_5c_6 \\ -c_5s_6 & c_4c_6 - s_4s_5s_6 & s_4c_6 + c_4s_5s_6 \\ s_5 & -s_4c_5 & c_4c_5 \end{pmatrix} \quad (1)$$

where $c_r = \cos x_r$, $s_r = \sin x_r$, and elements of the first row present projections of the unit vector on axis O_1x_1 etc.

Introducing radius vector $\vec{\rho}$ of the point O_1, vectors $\vec{h}, \vec{A}, \vec{R}_1, \vec{a}, \vec{H}$ forming the closed polygon for the upper part of magnetically interacting loops (see Figure 1) and similar vector polygon for the lower part of magnetically interacting loops (not depicted in Figure 1), and using (1) for the components of the vectors connecting arc's elements of rings and dipole centers after transformations, one can obtain

$$R_{j1} = X - (-1)^j ha_{31} + A\left(a_{11}\cos t_{ij} + a_{21}\sin t_{ij}\right),$$

$$R_{j2} = Y - (-1)^j ha_{32} + A\left(a_{12}\cos t_{ij} + a_{22}\sin t_{ij}\right),$$

$$R_{j3} = Z - (-1)^j ha_{33} + A\left(a_{13}\cos t_{ij} + a_{23}\sin t_{ij}\right) + (-1)^j H \quad (2)$$

where index $j = 1$ is applied to the upper and $j = 2$ to the lower parts of loops, parameter h is a non-dimensional half-distance between rotor's loops, H is the same for the immobile rings, and the angle t_{ij} together with non-dimensional shift A determine a dipole-place on the rotor.

The mutual inductance between a ring and a dipole can be derived as the magnetic flux generated by the unit electric current in the corresponding ring and piercing this dipole area. Assuming that each dipole plane is perpendicular to the rotor axis and its area equals S, taking into account known formulae for the ring magnetic induction (see e.g. [9]) and (2), for the non-dimensional ring-dipole mutual inductance the following formula takes place

$$L_j = \left(b_{j1}\left(a_{31}R_{j1} + a_{32}R_{j2}\right) + b_{j2}a_{33}\right) \quad (3)$$

where

$$b_{j1} = \frac{R_{j3}}{r_{j1}}\left(-K(k_j) + \frac{1 + r_{j1}^2 + R_{j3}^2}{(1 - r_{j1})^2 + R_{j3}^2}E(k_j)\right)$$

$$b_{j2} = K(k_j) + \frac{1 - r_{j1}^2 - R_{j3}^2}{(1 - r_{j1})^2 + R_{j3}^2}E(k_j) \quad (4)$$

$$r_{j1} = \sqrt{R_{j1}{}^2 + R_{j2}{}^2}$$

where $K(k_j)$, $E(k_j)$ are full elliptic integrals of the modulus

$$k_j = 2\sqrt{r_{j1}\left((1 + r_{j1})^2 + R_{j3}^2\right)^{-1}}\,. \quad (5)$$

Any non-dimensional ring-dipole inductance L_0 corresponding to the MPW-position is determined by zero values of all non-dimensional degrees of freedom ($X = Y = Z = x_4 = x_5 = x_6 = 0$) in (3).

On the basis of analytical electromechanics [10], the magnetic energy represented as a function of the magnetic linkages and mechanical coordinates (degrees of freedom) is the potential energy of an electromechanical dynamic system. Therefore, with taking into account the said suppositions and smallness in sizes of dipoles loops, one can derive the formula for the potential energy u in the non-dimensional form

$$u = UU_0^{-1} = \sum_{j=1}^{2}\frac{\Psi_j^2}{\Psi_2^2}\sum_{i=1}^{N}(L_0 - L_{ij})^2 + U_1 Z \quad (6)$$

where characteristic magnetic energy U_0 and non-dimensional gravity energy U_1 are respectively

$$U_0 = \frac{1}{L}\left(\frac{\mu_0 S}{2a\pi}\frac{\Psi_2}{L_0}\right)^2, \; U_1 = mgaL\left(\frac{\mu_0 S}{2a\pi}\frac{\Psi_2}{L_0}\right)^{-2}. \quad (7)$$

III. THE STABILITY PROBLEM

One of essential problems is the stability problem for either equilibrium or spinning of a levitated rotor. This problem can be investigated without corresponding dynamic equations analysis on the basis of the Lyapunov's theorem about partial stability [11]. This problem is reduced to finding the positivity conditions for the potential energy expanded into the Taylor series in corresponding variables. In our case using Maple [12], the Taylor series for N= 4 dipole can be written as

$$u = B_1\left(X^2 + Y^2\right) + B_2 Z^2 + B_3(x_4^2 + x_5^2). \qquad (8)$$

where expressions for values B_i (i=1,..,3) are too cumbersome functions of the geometrical parameters and here are omitted. In accordance with Lyapunov's theorem, the sufficient conditions of the free equilibrium stability are equivalent to positive all parameters B_i. As simulations show, this requirement can be satisfied by a relevant choice of the system parameters.

IV. THE DYNAMICS MODEL

The potential energy determined above and Euler equations of a free body dynamics [8] allow deriving the starting equations of the rotor motion in the non-dimensional form (function u is determined by (6))

$$\frac{d^2X}{dt^2} = -A_1\frac{\partial u}{\partial X},$$

$$\frac{d^2Y}{dt^2} = -A_1\frac{\partial u}{\partial Y},$$

$$\frac{d^2Z}{dt^2} = -A_1\frac{\partial u}{\partial Z},$$

$$\frac{dn_1}{dt} + k_1 n_3 n_2 = -A_2\frac{\partial u}{\partial x_4},$$

$$\frac{dn_2}{dt} - k_1 n_1 n_3 = -A_2\frac{\partial u}{\partial x_5},$$

$$\frac{dn_3}{dt} = 0, \qquad (9)$$

$$n_1 = \cos x_5 \cos x_6 \frac{dx_4}{dt} + \sin x_6 \frac{dx_5}{dt},$$

$$n_2 = -\cos x_5 \sin x_6 \frac{dx_4}{dt} + \cos x_6 \frac{dx_5}{dt},$$

$$n_3 = \sin x_5 \frac{dx_4}{dt} + \frac{dx_6}{dt}.$$

Here non-dimensional parameter $A_1 = U_0\left(ma^2w^2L\right)^1$ is ratio between magnetic characteristic energy U_0 and characteristic kinetic energy (w is the characteristic angular velocity); non-dimensional parameter $A_2 = U_0(I_1w^2L)^{-1}$ determines ratio between magnetic characteristic energy U_0 and rotating kinetic energy with inertia moment I_1 relatively axis O_1x_1 or O_1y_1; non-dimensional parameter $k_1 = \left(I_3 - I_1\right)I_1^{-1}$ is ratio of the rotor main central inertia moments (I_3 is the inertia moment relatively axis O_1z_1); n_1, n_2, n_3 are non-dimensional angular velocity components on axis O_1x_1, O_1y_1, and O_1z_1 respectively, and t is the non-dimensional time.

Expressions for partial derivatives of the non-dimensional potential energy u in (9) are too cumbersome and therefore omitted.

The conservative 12[th]-order system (9) with potential energy determined above is too complicated to be analyzed by known analytical methods. In this case the software Maple11 [12] is more relevant.

We have developed Maple-based tools that give full realization of the free rotor dynamics. As an example, we represent numerical solutions for the Cauchy problem and phase portraits building. The plots of solutions and phase portraits shown in Figures 2-5 depict the case of assumed $H - h = 0.1$, $A_1 = 1$, $A_2 = 1$, $k_1 = 0$, $U_1 = 0$ and the following initial conditions: $X(0) = 0.05$, $\dot{X}(0) = 0$, $Y(0) = 0$, $\dot{Y}(0) = 0.01$, $Z(0) = 0.01$, $\dot{Z}(0) = 0$, $x_4(0) = 0$, $x_5(0) = 0$, $x_6(0) = 0$, $n_1(0) = 0$, $n_2(0) = 0$, $n_3(0) = 100$.

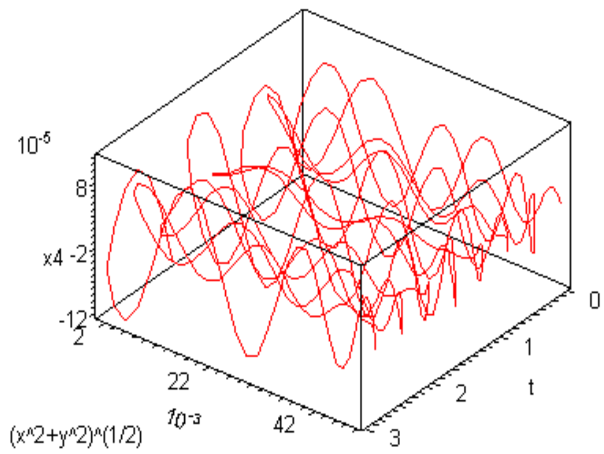

Fig. 2. Phase portrait t, $\sqrt{X(t)^2 + Y(t)^2}$, $x_4(t)$.

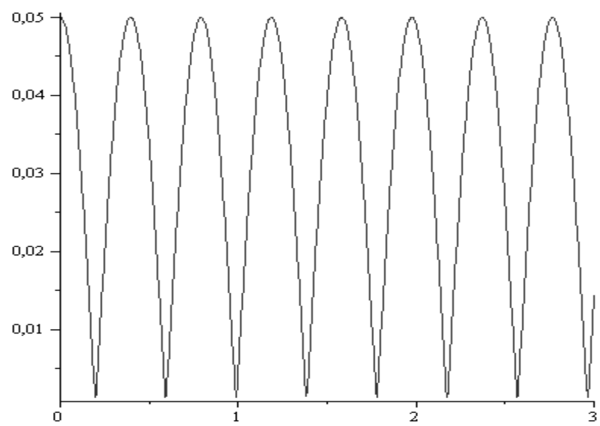

Fig. 3. Radial separation versus time.

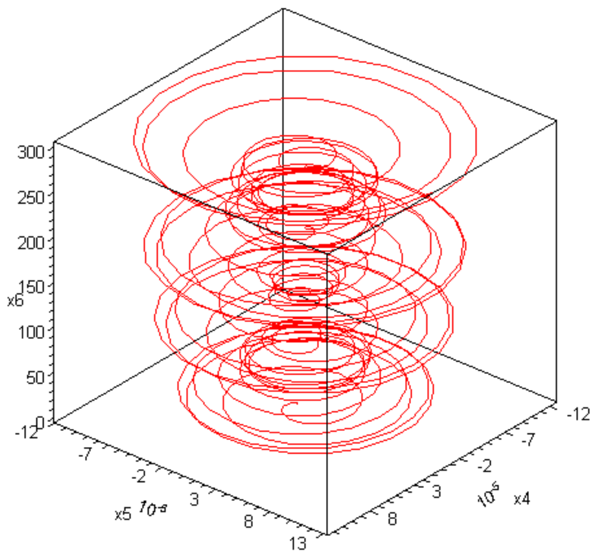

Fig. 4. Phase portrait of x_4, x_5, x_6 .

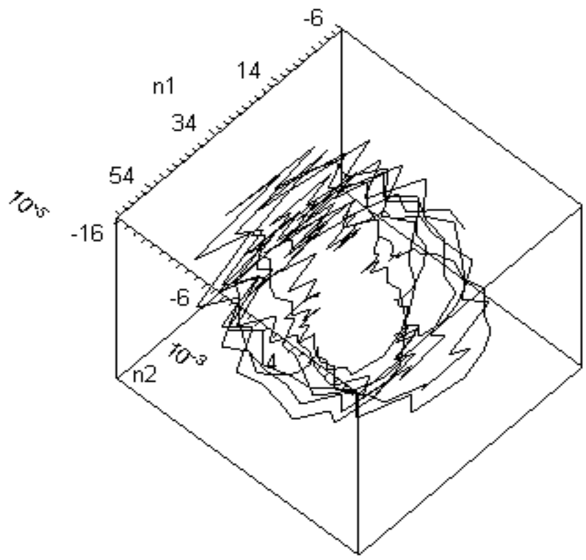

Fig. 5. Phase portrait of n_1, n_2, n_3 .

V. CONCLUSIONS

The free rotor dynamics problem is considered in the parts of mathematical model construction, substantiation of the free equilibrium stability (i.e. levitation), and dynamic analysis on the basis of the modern computer tools. The mathematical model taking into account six degrees of freedom proved to be complicated non-linear conservative dynamic system of the 12th-order, an analytically unsolvable problem.

The free equilibrium stability i.e. levitation derived on the Lyapunov's theorem is possible by the MPW-manifestation and relevant choice of the magnetic and geometrical parameters.

As examples demonstrate, the software Maple proved to be the effective computer tool to analyze the free rotor dynamics. This approach can be useful to develop superconducting bearings [13-16].

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